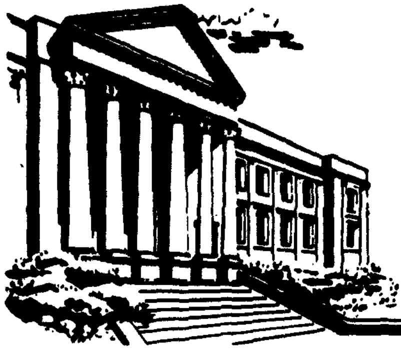

FIRST INTERIM TECHNICAL REPORT I-A2217-1

RESEARCH IN SURVEYING, MAPPING, AND GEODESY

REFRACTION IN SELECTED MODEL ATMOSPHERES. III.

by

James K. Gleim
Robert S. Grubmeyer
John E. Merrill
William M. Protheroe

February 12, 1963

Contract DA-44-009-ENG-3767
Department of the Army Project No. 8T35-12-001-01

Placed by

U. S. ARMY ENGINEER
Geodesy, Intelligence and Mapping Research and Development Agency
Fort Belvoir, Virginia

THE FRANKLIN INSTITUTE

LABORATORIES FOR RESEARCH AND DEVELOPMENT

PHILADELPHIA PENNSYLVANIA

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PREFACE

This project was initiated by the U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency (GIMRADA). It is being pursued under Contract DA-44-009-ENG-3767, "Research in Surveying, Mapping and Geodesy," authorised by and negotiated under Title 10, U. S. C., Section 2304 (a) (1) and Presidential Proclamation 2914.

Starting date of the original contract was September 4, 1958. The work reported herein was done mainly between October 1961 and November 1962 under subsequent modifications (in effect Numbers 5 and 8) of that contract. Terminal date of Modification No. 8, presently in force, is June 30, 1964. Execution of the project is under the direction of Mr. Nathan Fishel of GIMRADA, Contracting Officer's Technical Representative.

This project, as a concern of the Operations Analysis Laboratory of the Franklin Institute Laboratories, has been under the supervision of Mr. Robert S. Grubmeyer, Manager of that Laboratory. Mr. James K. Gleim and Dr. John E. Merrill carried out the study reported here; Dr. William M. Protheroe served as consultant on various aspects of the problem.

In accordance with the pattern established in Modification No. 8 of the basic contract, this report is designated Interim Technical Report 1. As a theoretical-computational investigation of the relation between the model chosen to represent the earth's atmosphere at a given time, and the amount by which an incoming ray of light would be deviated from its original direction, it is closely related to the study of the 1959 ARDC model presented in our Quarterly Report No. 6 and that of more generalised atmosphere models presented in Part I of our Quarterly No. 7; for this reason the present report has III appended to the newly-adopted sub-title.

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SUMMARY

This report continues the investigation of the astronomical refraction in spherically-stratified model atmospheres. Previously, the study of the refraction in the 1959 ARDC model was presented [1]. Here we investigate the astronomical refraction in tropical and arctic winter atmosphere models at zenith distances of 60° and 85°. The procedure is similar to that used in the study of the 1959 ARDC model; the results, given in the following table, show satisfactory agreement with their respective values computed from the Willis and Pulkovo tables.

Atmosphere Model	$z = 60^\circ$	$z = 85^\circ$
Tropical	93."043	542."292
Arctic Winter	116.944	700.787

Also included is a sample calculation for a homogeneous atmosphere, considered as a limiting case. Again the results show good agreement with Pulkovo, to rather considerable zenith distances.

The effect of injecting water vapor into the dry 1959 ARDC model was examined. The procedure consisted in revising the value of the index of refraction at each level where necessary, using the Barrell-Sears expression for n. The results indicate that for a zenith distance of 60° there is no difference in the contribution to the total refraction between the dry and humid models above 15 km altitude, and for a zenith distance of 85°, there is no difference between the two models above 30 km.

I. INTRODUCTION

This report discusses work on the problem of atmospheric refraction beyond that reported in Quarterly Report, Q-A2217-6[1]. The main portion deals with the investigation of the amount of refraction caused by atmospheres based upon tropical and arctic winter models; a simplified calculation compares the refraction produced by a homogeneous atmosphere with that based on a realistic model; and a preliminary discussion of the change in refraction caused by introducing water vapor into the ARDC model[2] is also included.

The data for the tropical and arctic winter (or polar) models were obtained from tables in "Military Standard 210 A, Climatic Extremes for Military Equipment," published by the Department of Defense, 2 August 1957. The important features of these tables are:

1. The data are given to an altitude of 100,000 feet
2. Sea-level temperature is + 32.114°C for the tropical model and - 27°C for the arctic winter atmosphere
3. Sea-level pressure is 760 mm Hg and 768.8062 mm Hg for the two models, respectively
4. The value of gravity is assumed constant throughout both atmospheres, (980.665 cm/sec^2), and
5. The composition of both atmospheres is assumed constant throughout.

The tropical atmosphere represents an annual mean day between 0° and 20° N latitude, while the arctic winter profile is based on radiosonde information obtained at seven North American stations north of latitude 60° N.

For our purposes both atmospheres were assumed to be valid at the latitude for which the ARDC atmosphere was constructed, $45^{\circ}32'33''$ N.

The procedure for integration was similar to that used for the ARDC model, and is discussed in detail in Q-A2217-6 and in Section II. A. below. The main problem with these atmospheres is that the tabulations extend, for perfectly good meteorological reasons, to an altitude of only 30 km. This necessitated the grafting of an ARDC top onto each, in order to insure that all the atmospheric layers which might contribute to the refraction were included in the integration. The technique for doing this is also given in Section II. A. of this report.

Comparisons with values from the Willis [3] and Pulkovo [4] tables were made and the results and analyses of these are included in a later section.

II. INTEGRATION OF THE TROPICAL ATMOSPHERE MODEL

A. Mechanics of the Integration

The procedure for the integration of the tropical atmosphere was similar to that previously used in the ARDC case. The basic equation to be integrated is (from Q-A2217-6, page 3)

$$R'' = 206265 \sin z \int_{N=\frac{1}{n_o}}^{N=1} \frac{dN}{N\left[(1 + h/a)^2 N^2 - \sin^2 z\right]^{\frac{1}{2}}} \qquad (1)$$

The value of "a" was taken as 20890722.13 feet and the zenith angles for which the refractions were computed were 60° and 85°, the same as for the ARDC model. N (the ratio of the index of refraction at any height to the index of refraction at the surface) was calculated for thirty-five levels, selected at equal intervals of 3000-foot pressure heights by the following formula, based on the Barrell and Sears expression for n at any temperature and pressure [5]:

$$N_T = 0.9997382825 + \frac{0.3846928729\left[p + (104900 - 157t)\,p^2 \times 10^{-6}\right]}{(1 + .003661t)10^6} \qquad (2)$$

where t = temperature ($^{\circ}$C) and p = pressure (mm Hg) obtained from the tropical atmosphere table of Military Standard 210 A. The table actually gives p in inches of mercury, but this was converted to mm Hg in order to make the expression for N compatible with that previously derived.

In order to insure that no significant amount of refraction was omitted and to make comparisons with other models more convenient, an extension from the 100,000-foot top of the tropical atmosphere table, to 90 km, was made. This was done, in effect, by grafting the ARDC atmosphere model from 30.627 km to 90 km onto the tropical atmosphere. Two assumptions were made: (1) as before, the amount contributed to the total refraction by any reasonable atmospheric model above 90 km is negligible and (2) the upper part of the ARDC atmosphere model is a very close approximation to what the upper part of the tropical atmosphere model would be if it existed. This procedure requires a knowledge of the refraction effect at the interface where the two models were joined together.

In order to present the description of the procedure followed as clearly as possible, we must introduce a few symbols for notation. Let A refer to the ARDC atmosphere, which was reported in Quarterly Report No. 6, T to the tropical model, TA to the tropical model above 30 km, AW to the arctic winter atmosphere model, and AWA to the arctic winter model above 30 km. These will be used principally as subscripts; other minor notations will be introduced as necessary. Also, henceforth in this report "Tropical Atmosphere Model" will be taken to mean the profile based on the data from the Military Standard 210 A tropical atmosphere table up to 30 km, plus a top based on the 1959 ARDC profile from 30 km to 90 km. Similarly, the "Arctic Winter Atmosphere Model" will be taken to mean the profile based on the data

from the Military Standard 210 A polar atmosphere table up to 30 km, plus a top based on the 1959 ARDC profile from 30 km to 90 km.

The first problem, that of integrating the ARDC model above 30 km, using the tropical sea-level parameters was accomplished as follows. In Quarterly Report No. 6 the integration of the ARDC atmosphere was expressed in terms of the variable $N_A = n_h/n_{OA}$ where n_h is the index of refraction at height h, and n_{OA} is the index of refraction at the base of A. N_A occurs not only in the numerator (dN_A), but also inside and outside the brackets in the denominator in equation (1). The integrand of this expression was altered for the present case, by putting

$$N_{TA} = N_A \times \frac{n_{OA}}{n_{OT}} = N_A \times 1.0000156009 \qquad (3)$$

and $$(1 + h/a)^2 N_{TA}^2 = (1 + h/a)^2 N_A^2 \times 1.0000312022 \qquad (4)$$

since $$n_{OA} = 1.000277391$$

and $$n_{OT} = 1.000261786$$

by equations (3,4) of Quarterly Report No. 6. The integration for the upper part of the tropical atmosphere was actually done for 35 to 90 km only. The interval from 30.627 km (100,000 ft) to 35 km is a transition interval treated as explained below.

The second problem, refraction by the transition zone, was examined in three different ways. The agreement of the results indicates that any one of the three is a satisfactory approach to our problem.

The first, and most obvious approach, is by means of Snell's law. In Figure 1 let i_T be the angle that the outgoing ray makes with the vertical at the transition zone, treated as a thin shell or interface of constant index n_T. Then $n_{OT} r_O \sin z = n_T r_1 \sin i_T$. Similarly, if i_A is the angle the outgoing ray makes with the vertical at the interface, assuming an index n_A on the upper side of the surface, then $n_{OT} r_O \sin z = n_A r_1 \sin i_A$ and therefore $n_T \sin i_T = n_A \sin i_A$ which is the ordinary law of refraction at an interface between media with indices of refraction n_T and n_A. Solution of these equations results in a value of $0\overset{''}{.}0131$ for $i_T - i_A$ at $z = 60^\circ$ and $0\overset{''}{.}0588$ at $z = 85^\circ$. These quantities represent the amount by which the outgoing ray refracts <u>towards</u> the normal in crossing the interface.

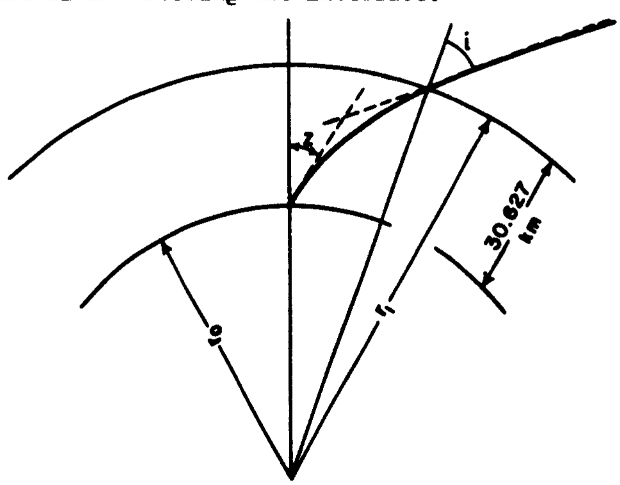


FIG. I. INTERFACE GEOMETRY

The second method again considers the transition zone as an extremely small interval, that is, an interface, the lower surface of which is the top of the tropical, and the upper surface, the bottom of the TA atmospheric model. This in effect puts $h_T = h_A$ and hence $p_T = p_A$ since the tropical atmosphere table is given with ICAO pressure height as an argument. However, $T_A \neq t_T$ and therefore $n_A \neq n_T$ and $N_A \neq N_T$. In fact, $N_T = \frac{n_{hT}}{n_{OT}} = 0.9997419033$ and $N_{AT} = \frac{n_{hA}}{n_{OT}} = 0.9997419412$. This procedure results in refractions in the amount of $0\overset{''}{.}0133$ for $z = 60^\circ$ and $0\overset{''}{.}0602$ for $z = 85^\circ$. Note further that since $n_{hA} > n_{hT}$, the ray traced outwards bends <u>toward</u> the vertical in crossing the interface, as previously concluded. Thus, in effect, a negative increment is introduced into the summation of the refraction integral.

The third procedure joins the atmospheres at a point where the temperatures come together. This involves an extrapolation of the temperature gradient of the tropical atmosphere and assumes that both gradients are linear when expressed as a function of geopotential feet. This is not quite true, but the deviation is negligible for our purposes. The pressures at this altitude will also be equal, hence the n's and N's. With these assumptions the temperatures were found to come together at 34.858 geopotential km (or 35.05 geometric km). The common temperature at this level is - 26.93°C and the ICAO pressure is 4.348 mm Hg. An N for this level was calculated and the computation of the refraction for the 30.627 to 35.050 km, 35.050 to 40 km, and the

40 to 90 km intervals constitutes the third method of grafting the two atmospheres together. The results for this procedure, for both the 60° and the 85° zenith angle cases, are given in Section B.

B. Results

The refraction produced by an atmosphere of the model assumed, and computed in the manner described in Section II.A. is given in Table 1 for the two zenith distances, 60° and 85°. The left side of the table presents the refraction, accumulated from the effective top of the TA model, down to any level, h, terminating at the upper side of the interface. The right side of the table presents the refraction, accumulated from the top of the tropical atmosphere proper down to any level h. The entries in the interval column were obtained by converting the entries, taken at 3000-foot intervals in the altitude column of the tropical atmosphere table into kilometers (1 ft. = .3048 meters exactly, ICAO Extension, 1959). Also included in this table are values of the refraction at the interface calculated by the third method described previously. Table 2 summarizes the results for the integration of the tropical atmosphere.

Table 1

REFRACTION ACCUMULATED DOWN TO ANY LEVEL, h,
IN THE TROPICAL ATMOSPHERE MODEL

Level h (km)	z = 60°	z = 85°
90 - 85	0″.0003	0″.0009
80	.0013	.0043
75	.0034	.0114
70	.0075	.0257
65	.0147	.0513
60	.0271	.0966
55	.0471	.1717
50	.0842	.3148
45	.1582	.6099
40	.3143	1.2539
35	.6509	2.6953
30.627	1.2783	5.4849
40-35.05	0.3319	1.4208
35.05-30.627	0.6188	2.7499

Level h (km)	z = 60°	z = 85°
30.7-30.4	0″.0604	0″.2736
29.5	.2711	1.2336
28.5	.5110	2.3362
27.6	.7969	3.6607
26.7	1.1324	5.2293
25.7	1.5326	7.1165
24.8	1.9981	9.3320
23.9	2.5474	11.9703
23.0	3.1802	15.0381
22.1	3.9266	18.6906
21.2	4.8107	23.0580
20.3	5.8895	28.4380
19.4	7.1571	34.8202
18.5	8.6474	42.3947
17.7	10.4041	51.4076
16.9	12.4694	62.1042
16.1	14.2642	71.4882
15.2	16.1856	81.6337
14.3	18.3461	93.1598
13.4	20.7690	106.2266
12.5	23.4809	121.0185
11.6	26.5452	137.9334
10.6	29.9191	156.7897
9.6	33.6107	177.6871
8.7	37.6293	200.7345
7.7	41.9815	226.0315
6.8	46.7094	253.8904
5.8	51.8293	284.4846
4.8	57.3296	317.8255
3.9	63.2851	354.4587
2.9	69.6666	394.3059
1.9	76.5239	437.7861
1.0	83.9065	485.3395
0.0	91.7782	536.8664

Table 2

SUMMARY OF RESULTS, NUMERICAL INTEGRATION
OF THE TROPICAL ATMOSPHERE MODEL

z = 60°

Level (km)	Method a	Method b	Level (km)	Method c
90-30.627	1".2783	1".2783	90 - 40	0".3143
Interface	- 0.0131	- 0.0133	40 - 35.05	0.3319
30.627-0	91.7782	91.7782	35.05 - 30.627	0.6188
			30.627 - 0	91.7782
Totals	93".0434	93".0432	Total	93".0432

z = 85°

Level (km)	Method a	Method b	Level (km)	Method c
90-30.627	5".4849	5".4849	90 - 40	1".2539
Interface	- 0.0588	- 0.0602	40 - 35.05	2.7499
30.627-0	536.8664	536.8664	35.05 - 30.627	1.4208
			30.627 - 0	536.8864
Totals	542".2925	542".2911	Total	542".2910

Method a - Snell's law
Method b - Interface as a thin layer
Method c - Extrapolation of temperature gradient

C. Comparisons

Table 3 shows the comparisons of the refraction obtained by the integration of this model atmosphere, with that obtained from the Willis and Pulkovo systems, using the same values for the parameters at the base. A discussion of these results will be postponed until the arctic winter data are added in a later section of the report.

Table 3

COMPARISONS OF THE REFRACTIONS: TROPICAL INTEGRATION, WILLIS, AND PULKOVO

	z = 60°	z = 85°
Tropical	93".043	542".292
Willis	92.990	542.678
Pulkovo	92.946	542.062

III. INTEGRATION OF THE ARCTIC WINTER ATMOSPHERE MODEL

A. Mechanics of the Integration

The procedure for the integration of the arctic winter atmospheric model was the same as that for the tropical model. The sea-level values of the temperatures and pressure are -27°C and 768.8072 mm Hg, respectively. The AW atmosphere was assumed to be valid for the latitude of the ARDC model; hence the value of the earth's radius was unchanged. The value of N_{OAW}, 1.000328653, was computed from equation (4) of Quarterly Report No. 6. The $N_{AW's}$ were computed from the following expression:

$$N_{AW} = 0.9996714550 + \frac{0.3846671581 \left[p + (1.049 - .0157t)10^{-6}p^2\right]}{(1 + .003661t)10^6} \quad (5)$$

An ARDC top was computed for the AW model, in much the same way as for the tropical model. In order to equalize the pressures in the two atmospheres, it was necessary to lower the ARDC levels, in effect, by 1.3 km resulting in a new set of $N^{'}$s,

$$N_{AWA} = \frac{n_{OA}}{n_{OAW}} = (0.9999487548)\ N_A \quad (6)$$

The interface of this combined atmospheric model is at 29.337 geometric kilometers, or 96,249 geometric feet (as given by the extreme right-hand column of the arctic winter atmosphere table). The refraction at this level was computed by the first two methods previously used, namely by

Snell's law and by the introduction of a small, thin layer. The first procedure resulted in refractions of $0''.1412$ and $0''.6432$ for $z = 60^\circ$ and $z = 85^\circ$ respectively, while the second method resulted in refractions of $0''.1411$ and $0''.6490$ for $z = 60^\circ$ and $z = 85^\circ$ respectively. In all cases, the ray traced outwards from the observer bends <u>away</u> from the normal in crossing the interface, opposite to the situation found for the tropical-ARDC model.

B. Results

The refraction produced by an atmosphere of the model assumed, and computed in the manner described in Section III.A., is given in Table 4 for the two zenith distances, 60° and 85°. The left side of the table presents the refraction, accumulated from the effective top of the AWA model, down to any level, h, terminating at the upper side of the interface. The right side of the table presents the refraction, accumulated from the top of the arctic winter atmosphere proper down to any level, h. Also included in this table is the value of the refraction at the interface calculated according to the second method described above. Table 5 summarizes the results for the integration of the arctic winter atmosphere.

Table 4

REFRACTION, ACCUMULATED DOWN TO ANY LEVEL, h, IN THE ARCTIC WINTER ATMOSPHERE MODEL

Level h (km)	z = 60°	z = 85°	Level h (km)	z = 60°	z = 85°
88.7-83.7	0".0003	0".0011	29.3-29.1	0".0650	0".2994
78.7	.0014	.0045	28.2	.2797	1.3004
73.7	.0034	.0115	27.4	.5294	2.4652
68.7	.0075	.0259	26.5	.8215	3.8386
63.7	.0147	.0518	25.7	1.1612	5.4494
58.7	.0272	.0975	24.8	1.5532	7.3238
53.7	.0472	.1734	23.9	2.0095	9.5249
48.7	.0843	.3183	23.0	2.5362	12.0888
43.7	.1584	.6171	22.1	3.1414	15.0619
38.7	.3147	1.2700	21.2	3.8427	18.5396
33.7	.6516	2.7324	20.3	4.6500	22.5820
29.337	1.2797	5.5669	19.4	5.5838	27.3039
			18.5	6.6554	32.7774
Interface	+0.1411	+0.6490	17.6	7.8956	39.1769
			16.7	9.3262	46.6374
			15.8	10.9720	55.3120
			14.9	12.8774	65.4651
			14.0	15.0723	77.2926
			13.1	17.5986	91.0615
			12.2	20.5195	107.1669
			11.3	23.8910	125.9776
			10.4	27.7664	147.8621
			9.5	32.1823	173.1035
			8.6	37.0858	201.4762
			7.7	41.7170	228.6024
			6.8	46.7648	258.5358
			6.0	52.2772	291.6337
			5.1	58.2356	327.8642
			4.3	64.7189	367.7926
			3.4	71.7291	411.5262
			2.5	79.6504	461.5899
			1.7	89.1921	522.6707
			0.9	99.8071	591.4708
			0.1	113.9409	684.1398
			0.0	115.5229	694.5439

Table 5

SUMMARY OF RESULTS, NUMERICAL INTEGRATION
OF THE ARCTIC WINTER ATMOSPHERE MODEL

$z = 60°$

Level (km)	Method a	Method b
87.7 - 29.337	1".2797	1".2797
Interface	+ 0.1412	+ 0.1411
29.337 - 0	115.5229	115.5229
Total	116".9438	116".9437

$z = 85°$

Level (km)	Method a	Method b
87.7 - 29.337	5".5669	5".5669
Interface	+ 0.6432	+ 0.6490
29.337 - 0	694.5739	694.5739
Total	700".7840	700".7878

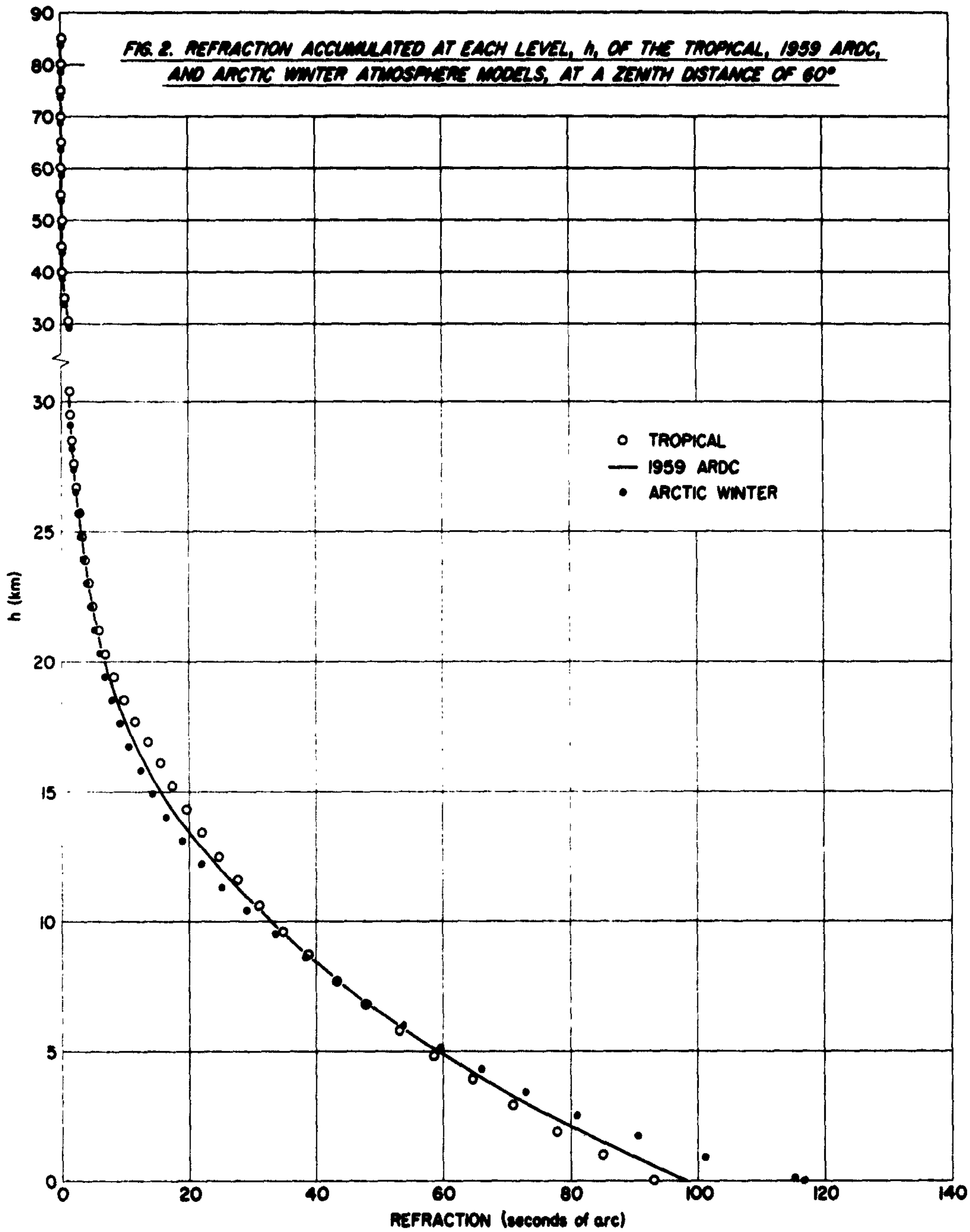
Method a - Snell's law
Method b - Interface as a thin layer

C. Comparisons and Analyses

Table 6 and Figures 2 and 3 summarize the refraction computations. In Table 6 the integrations of the three model atmospheres are compared with the corresponding values of the refractions obtained by the Willis and Pulkovo methods. To repeat, the same basic parameters were used in the Willis and Pulkovo methods as in the numerical integrations of the atmospheric models, namely:

1. Wavelength of light, .5753 microns
2. Standard gravity at latitude $45°32'33''$, 980.665 cm sec^{-2}
3. No water vapor in any of the models. In addition, the surface conditions for each atmosphere were as follows:

FIG. 2. REFRACTION ACCUMULATED AT EACH LEVEL, h, OF THE TROPICAL, 1959 ARDC, AND ARCTIC WINTER ATMOSPHERE MODELS, AT A ZENITH DISTANCE OF 60°



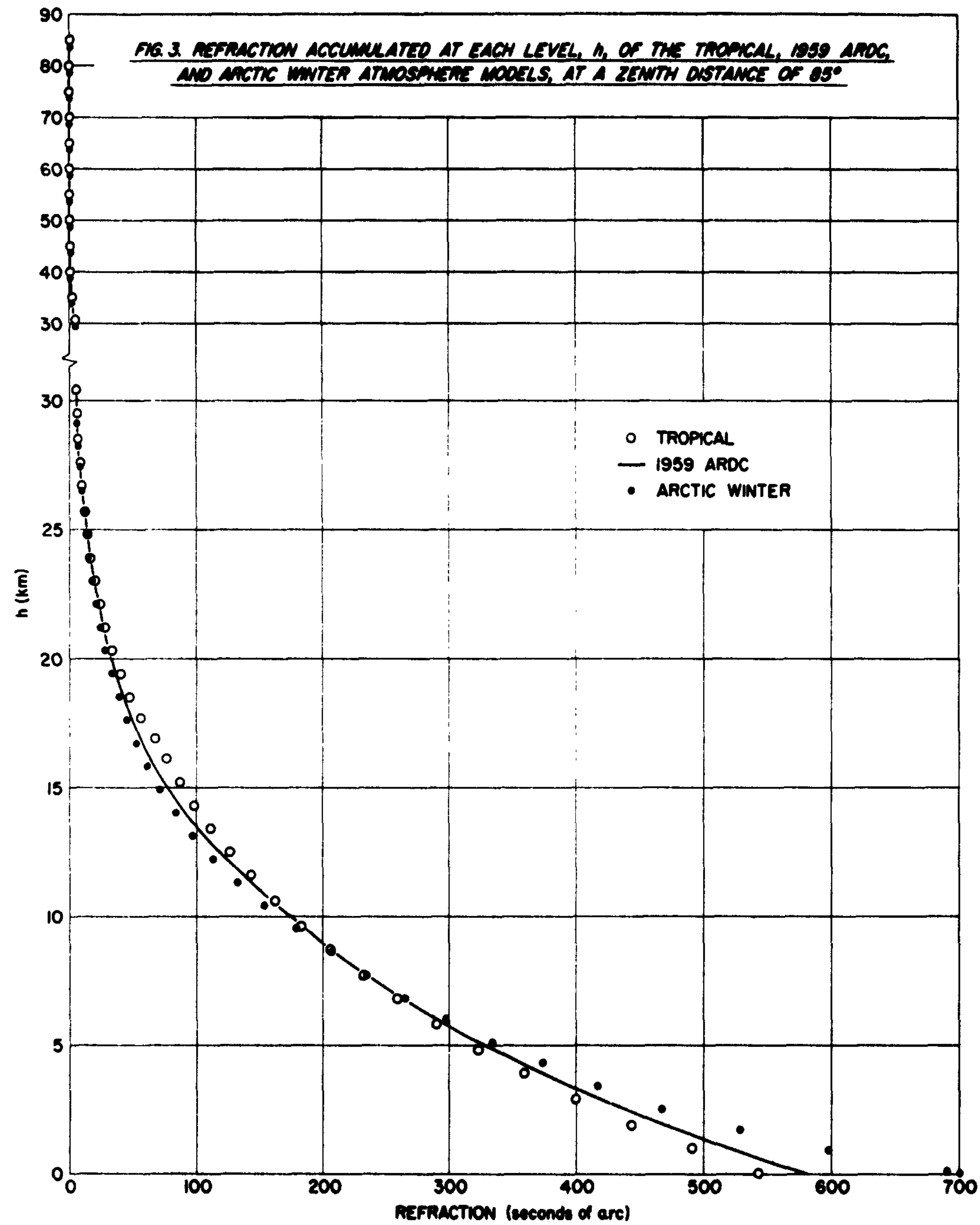


FIG. 3. REFRACTION ACCUMULATED AT EACH LEVEL, h, OF THE TROPICAL, 1959 ARDC, AND ARCTIC WINTER ATMOSPHERE MODELS, AT A ZENITH DISTANCE OF 85°
TROPICAL
1959 ARDC
ARCTIC WINTER
h (km)
REFRACTION (seconds of arc)
0
5
10
15
20
25
30
40
50
60
70
80
90
100
200
300
400
500
600
700

Atmospheric Model	T(^{o}C)	p (mm Hg)	$(n_o - 1) \times 10^6$
Tropical	+ 32.114	760	261.786
ARDC	+ 15	760	277.391
Arctic Winter	- 27	768.8072	328.653

At $z = 60^o$, the results are fairly consistent. The integration method in all three cases gives about $0\overset{''}{.}05$ more refraction than Willis. The integration also gives about $0\overset{''}{.}1$ more refraction than Pulkovo, except in the arctic winter case where it gives about $0\overset{''}{.}16$ more. Willis gives about $0\overset{''}{.}05$ more refraction than Pulkovo for the first two model atmospheres, and + $0\overset{''}{.}11$ more for the arctic winter atmosphere. One might place the blame for the deviation in the cold atmosphere case on the Pulkovo tables, but this is difficult to justify in view of the fact that these tables are observationally based, their standard conditions being representative of a high northern latitude. At a zenith angle of 85^o the numerical values are less well stabilized because any error is multiplied by a factor of almost ten (with respect to the data for $z = 60^o$) and because we are working at extreme limits in both tables, where even rounded-off decimals may be significant. For example, within the Willis tables, an uncertainty of $\pm\ 0\overset{''}{.}008$ can be expected in the value of the refraction at a zenith distance of 85^o, due primarily to the restriction of the number of decimals given for computing two of the basic parameters, f

and $\log_e n_o$. An uncertainty of 5×10^{-6} in the value of log R derived from the Pulkovo tables, gives an uncertainty of approximately the same order of magnitude in the refraction as that found for the Willis tables.

Table 6

SUMMARY OF REFRACTION COMPUTATIONS:
THE INTEGRATIONS OF THE THREE MODEL ATMOSPHERES
AND THE CORRESPONDING WILLIS AND PULKOVA VALUES

Method	Tropical	ARDC	Arctic Winter
		$z = 60^\circ$	
Integration	93".043	98".620	116".944
Willis	92.990	98.563	116.890
Pulkovo	92.946	98.512	116.784
I - W	+ ".053	+ ".057	+ ".054
I - P	+ .097	+ .108	+ .160
W - P	+ .044	+ .051	+ .106
		$z = 85^\circ$	
Integration	542".202	578".814	700".787
Willis	542.682	579.026	698.888
Pulkovo	542.138	578.512	698.933
I - W	- ".390	- ".212	+ 1".900
I - P	+ .134	+ .302	+ 1.855
W - P	+ .544	+ .514	- .045

At a zenith distance of 85°, Table 6 shows that the refraction obtained by integrating both the tropical and ARDC model atmospheres was less than that obtained by the Willis method and more than that obtained from the Pulkovo tables. The results, though not in as good agreement as those for a zenith distance of 60°, are consistent when allowance is made for the fact that at such a large zenith distance,

the effects of the small differences in the atmospheric models (those on which the tables are based) become highly magnified. Willis gives about $0\overset{''}{.}5$ more refraction than Pulkovo when the surface conditions are those of the tropical and ARDC atmosphere models. However, for the arctic winter atmosphere conditions, the integration gives nearly two seconds more refraction than either the Pulkovo or Willis methods, and the latter two come more into agreement, Pulkovo giving about $0\overset{''}{.}05$ more refraction than Willis.

In order to see how our three cases, two of which lie near the extremes in their sea level temperature parameters, fit into the over-all pattern of atmospheric refraction, a study was made of the refraction for the range of base conditions likely to be encountered on the earth's surface. The basic quantities on which these refraction computations were made are:

$t = +30, +15, 0, -15, -30 \quad (^{\circ}C)$

$p = 720, 760, 780 \quad$ (mm Hg)

$z = 60^{\circ}, 70^{\circ}, 80^{\circ}, 85^{\circ}$

Humidity: 0%

The results, shown as differences between the Willis and Pulkovo values of the refraction (in the sense Willis minus Pulkovo) are given in Table 7 and are plotted in Figure 4 as functions of the temperature for each zenith angle. The run of the differences forms a consistent

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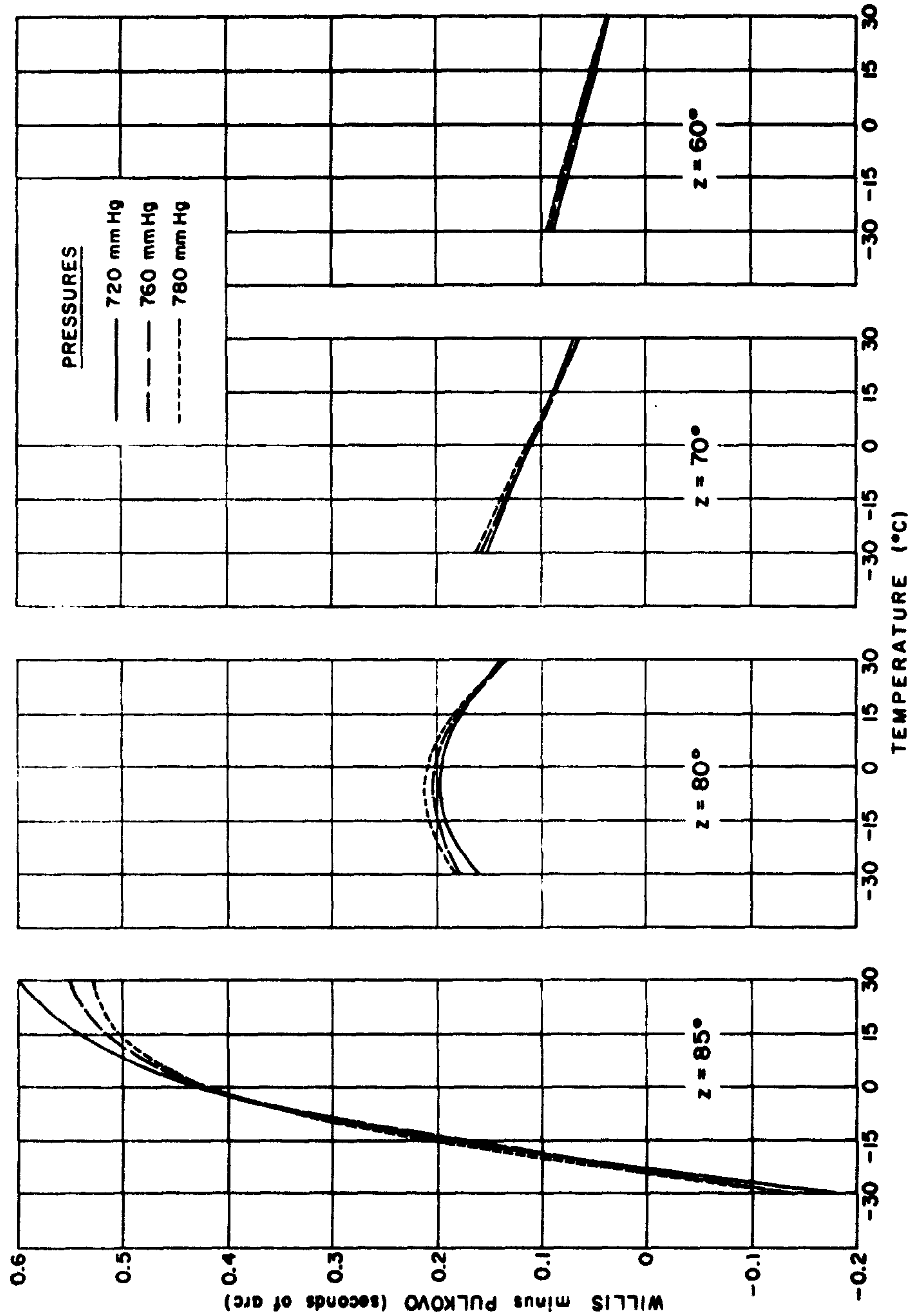

FIG. 4. COMPARISONS OF REFRACTIONS COMPUTED BY THE WILLIS AND PULKOVO METHODS

progression from z = 60° to z = 85°. However, the large range of the differences encountered at the zenith distance of 85°, makes it fairly clear that at large zenith distances one should realize that in speaking of orders of accuracy such as 0".001 or even 0".01, one must be very careful in making calculations with, and interpretations of, the numbers which are obtained from any existing method of refraction computation.

New model atmospheres have been recently, or are about to be, published. Any further study of refraction along the lines pursued here should naturally make use of them. Because of time limitations brought about by the pressure of other problems, there still remain some closely-related questions to be answered - for example, the effect of refraction at still larger zenith distances where the basic integral converges even more slowly for any realistic model.

Table 7

DIFFERENCES IN REFRACTION BASED ON THE WILLIS AND PULKOVO SYSTEMS, IN THE SENSE, W-P

t °C	p mm Hg	z = 60°	z = 70°	z = 80°	z = 85°
-30	720	+".089	+".154	+".161	-".180
	760	+.093	+.158	+.178	-.143
	780	+.095	+.164	+.183	-.128
-15	720	+.073	+.134	+.193	+.178
	760	+.077	+.131	+.199	+.188
	780	+.079	+.138	+.209	+.197
0	720	+.062	+.110	+.196	+.430
	760	+.065	+.111	+.200	+.418
	780	+.065	+.115	+.210	+.419
+15	720	+.048	+.087	+.179	+.540
	760	+.051	+.085	+.181	+.514
	780	+.052	+.090	+.184	+.503
+30	720	+.037	+.067	+.139	+.600
	760	+.037	+.062	+.133	+.549
	780	+.038	+.064	+.134	+.528

IV. REFRACTION BY A HOMOGENEOUS ATMOSPHERE

A very simple calculation was made to determine the magnitude of the refraction caused by a homogeneous atmosphere thought of as a limiting case. "Homogeneous" was taken to mean that the dependence of temperature, pressure and density on height was such that the refractive index remained constant throughout. The height of such an atmosphere, with a sea-level temperature of 15°C, is given by Allen [6] as 8.430 km. A series of calculations were made for various zenith angles based on the following formula derived by use of Snell's law:

$$R'' = \sin^{-1}\left(\frac{n_0\ a \sin z}{a + h}\right) - \sin^{-1}\left(\frac{a \sin z}{a + h}\right) \qquad (7)$$

where, as before, n_0 = index of refraction at the earth's surface, 1.000277391

a = radius of earth, 6367.49190 km

z = angle between the zenith and the direction of the ray at the observer

h = altitude, or in this case, the height of the homogeneous atmosphere, 8.430 km

The results are shown in Table 8, together with corresponding values computed from the Pulkovo tables. It appears, rather surprisingly, that down to an apparent zenith distance of 80°, even the extremely artificial homogeneous atmosphere provides a value of the refraction actually good enough for many ordinary purposes such as reduction of survey plates, and so forth.

Table 8

REFRACTION BY A HOMOGENEOUS ATMOSPHERE

Apparent z. d.	R_{Homog}	R_{Pulk}	H - P
0°	0"	0"	0"
10	10.076	10.063	+ 0.013
20	20.794	20.770	0.024
30	32.977	32.942	0.035
40	47.906	47.858	0.048
50	67.985	67.910	0.075
60	98.621	98.512	0.109
70	155.610	155.446	0.164
75	209.824	209.648	+ 0.176
80	312.397	312.479	- 0.082
85	570.897	578.512	- 7.615

V. THE EFFECT OF MOISTURE

The ARDC, tropical, and arctic winter model atmospheres are based upon the assumption of dry air. It is well known that humidity affects the astronomical refraction; both the Pulkovo and Willis methods of computation make allowance for the influence of moisture. This allowance is, however, based upon the moisture content of the air in the immediate vicinity of the observer. To test the correction for water vapor it was decided to introduce moisture into the ARDC model atmosphere at all levels and to re-integrate the refraction; this was then to be compared with the values predicted by the Willis and Pulkovo calculations. It was assumed that the moisture profile given by Murcray et al [7] is representative of the normal atmospheric conditions when moisture is present. Since large variations in the moisture content of the air within a few kilometers of the surface can exist, it was decided, for ease of computation, to use a profile consisting of four straight line segments to represent the mixing ratio of water to air by mass as a function of height. The assumed function and the original Murcray profile are shown in Figure 5.

The introduction of water vapor into the ARDC model entails a small adjustment in one or more of the three parameters, temperature, pressure, and density, describing the atmosphere as a function of height. This is caused by the modification in the mean molecular weight of the air brought about by the admixture of water. Since any change in density would in turn require a change in the pressure profile of the model,

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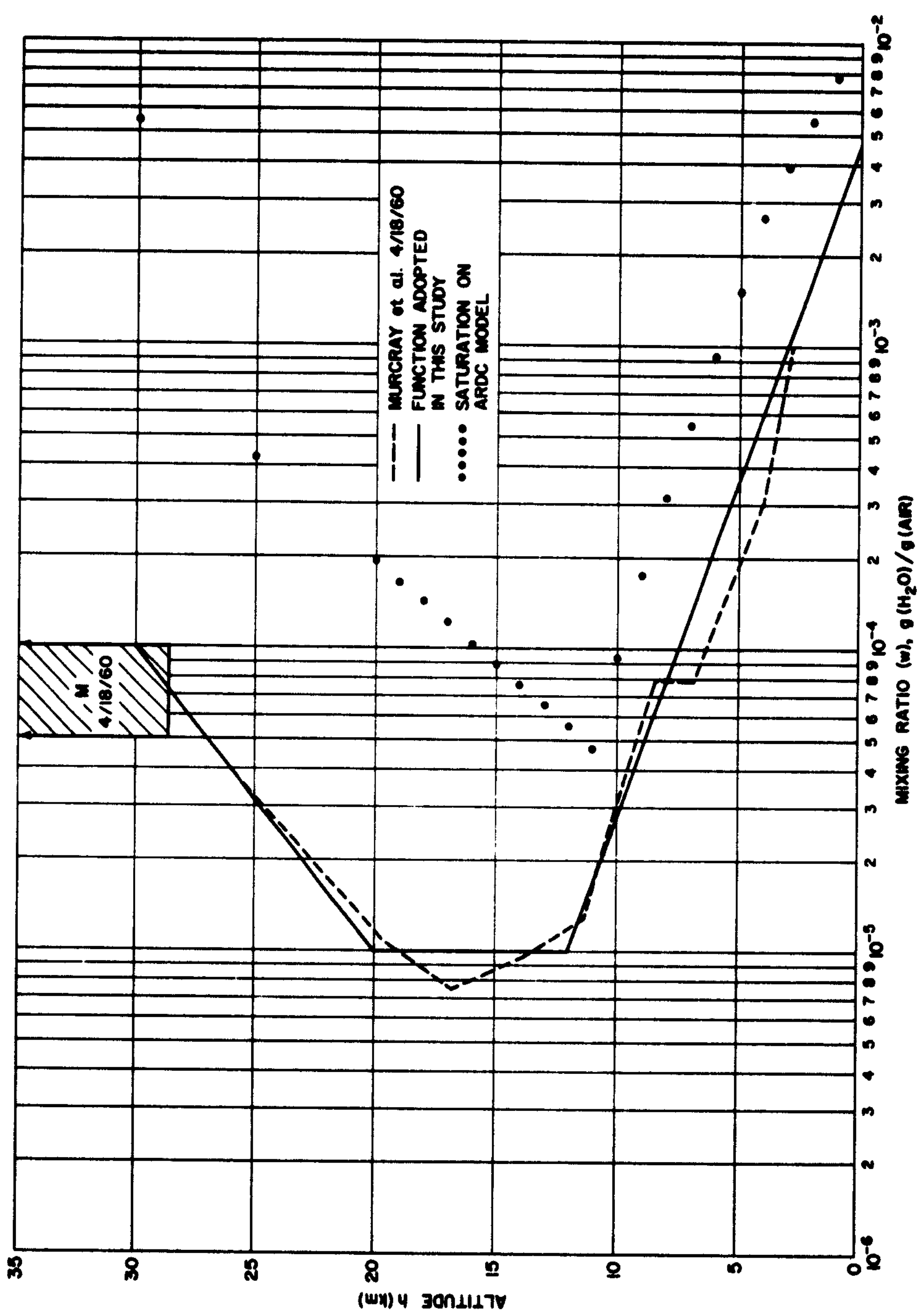

FIG. 5. THE ORIGINAL MURCRAY WATER VAPOR, ALTITUDE PROFILE AND THE WATER VAPOR, ALTITUDE PROFILE ADOPTED FOR THIS STUDY

it was decided to introduce the amount of water vapor required by the assumed mixing ratio profile in such a manner that the density of the moist atmosphere would remain the same as that tabulated in the dry ARDC model. This procedure, which retains the pressure profile of the dry ARDC and concentrates the adjustment into the temperature profile, seemed to be the one which would be likeliest to bring the tabulated ARDC model close to the reality of a humid atmosphere.

If the air is taken to obey the perfect gas law in accordance with the assumption used to construct the ARDC model, then the absolute temperature of the air at any level is given by

$$T = \frac{1}{R} \frac{P}{\rho} M \qquad (8)$$

where T = temperature in ^{o}K

P = the pressure in mm Hg

ρ = the density in grams/cc

R = the perfect gas constant in appropriate units

M = the mean molecular weight (dimensionless)

From the definition of mean molecular weight and the mixing ratio it is possible to write for each level

$$M = 28.966 - 17.520w \qquad (9)$$

where w = the mixing ratio in grams of H_2O per gram of air and 28.966 is the mean molecular weight of dry air for the composition assumed in the ARDC model.

The perfect gas law then gives

$$T = \frac{1}{R} \frac{P}{\rho} (28.966 - 17.520w) \tag{10}$$

The temperature of the dry ARDC atmosphere at any level can be written as

$$T_d = \frac{1}{R} \frac{P_d}{\rho_d} M_d \tag{11}$$

where the subscript d indicates the value of the quantity designated at the given level in the 1959 (dry) ARDC tables. Taking the ratio of the two expressions and noting that $\rho = \rho_d$ and $P = P_d$ by the assumption above and that $M_d = 28.966$, the temperature of the moist air may be written in terms of the "dry ARDC" temperature and the mixing ratio as

$$T = T_d (1 - 0.60485w) \tag{12}$$

This gives the value which the temperature of the moist air must assume at each level in order that its density and therefore its pressure at that level shall be the same as in the ARDC tables.

With the new, or modified temperature, it is necessary to recompute the refractive index at each level in the atmosphere; the new value was taken to be that given by the expression of Barrell and Sears for the refractive index of moist air, namely

$$(n - 1) = \frac{K_N p \ (1 + \beta_t p)}{1 + \alpha t} - \frac{f \quad (1 + 2\beta_t p) \ K_N - K'}{1 + \alpha t} \tag{13}$$

The quantities in this expression are defined as:

$$K_N = \left[0.378125 + 0.00214414\lambda^{-2} + 0.00001793\lambda^{-4}\right] \times 10^{-6}$$

$$K' = \left[0.3159 + 0.002963\lambda^{-2}\right] \times 10^{-6}$$

$$p = \text{total pressure in mm Hg}$$

$$\alpha = \text{constant} = 3.661 \times 10^{-3}$$

$$\beta_t = (1.049 - 0.0157t) \times 10^{-6}$$

$$t = \text{temperature, } ^{\circ}\text{Celsius}$$

$$\text{and } f = \frac{wp}{w + 0.62197}$$

The wavelength of the light was taken to be $\lambda = 0.5753$ microns as in our other studies of atmospheric refraction, giving the values

$$K_N = 0.38475875 \times 10^{-6}$$

and

$$K' = 0.32485247 \times 10^{-6}$$

The second term of the Barrel and Sears expression represents the direct influence of the water vapor upon the value of (n - 1). The value of (n - 1) is also modified indirectly at each level of the humidified model atmosphere, due to the modification in the temperature described above.

While it is possible to compute (n - 1) directly at each level from the Barrel-Sears formula, it may also be expressed as an expansion of this formula about the value at $t = t_d$, $w = f = 0$. This permits additional insight into the effects upon the index due to the two causes, change in temperature and inherent influence of water vapor upon refraction. Upon expansion and some simplification the expression for $(n_w - 1)$, the index of refraction for moist air, may be written as

$$(n_w - 1) = \left\{\frac{0.62197 - w}{0.62197 + w}\left[(n_d - 1) + \frac{K_N p^2 \gamma}{1 + \alpha t_d}\right] + \frac{f(K_N + K')}{1 + \alpha t_d}\right\}\left(1 + \frac{\Delta}{1 + \alpha t_d}\right) \qquad (14)$$

where n_d = the index of refraction for the given level in the ARDC model

$\gamma = 9.4961\ wT_d \times 10^{-9}$

$\Delta = 2.214356\ wT_d \times 10^{-3}$

and T_d = absolute temperature of the ARDC model at the corresponding height. This expression reduces to that used for dry air in our previous study [1] and is correct to much better than one percent for all reasonable values of w, t_d and p encountered in the problem.

An analysis of this expression indicates that the refractive index for the moist atmosphere will always be greater than that of the ARDC atmosphere for corresponding heights. The values of $(n_w - 1)$ were computed at selected heights and compared with the values of $(n_d - 1)$ already available at those heights. The results of the calculations are given in Table 9.

Table 9

COMPARISON OF INDICES OF REFRACTION AT SELECTED LEVELS IN THE MODIFIED (WET) ARDC ATMOSPHERE AND IN THE 1959 (DRY) ARDC ATMOSPHERE

	Wet ARDC		Dry ARDC	Difference
h	$w \times 10^3$	$(n_w - 1) \times 10^9$	$(n_d - 1) \times 10^9$	$(n_w - n_d) \times 10^9$
55	0.100	138.01	138	0.01
50	0.100	245.01	245	0.01
30	0.100	4,042.14	4,042	0.14
25	0.032	9,198.10	9,198	0.10
20	0.010	20,122.07	20,122	0.07
15	0.010	44,080.07	44,080	0.07
10	0.028	93,610.9	93.610	0.9
9	0.046	105,739.7	105,738	1.7
8	0.077	119,039.2	119,036	3.2
7	0.129	133,589.1	133,583	6.1
6	0.215	149,469.4	149.458	11.4
5	0.359	166,760.2	166,739	21.2
4	0.599	185,560.2	185,521	39.2
3	1.000	205,957.9	205,885	72.9
2	1.670	228,060.9	227,926	134.9
1	2.780	251,975.3	251,727	248.3
0	4.640	277,848.8	277,391	457.8

It is obvious from an inspection of the table that most of the change in refraction resulting from introduction of the water vapor occurs close to the surface (more than 70% of the change is within the lower 2 km) and that practically all significant change has occured

within the first half dozen or so kilometers. This may be taken as a reasonable explanation of the known fact that the "humidity corrections" used with the Pulkovo and Willis tables (though based only on the local surface humidity) are valid for most purposes. It is of course possible that a suitably, but slightly augmented value of n based upon the surface water vapor would correct the integrated refraction computed for dry air in most cases but would almost certainly tend to give divergent results for the refraction at large zenith distances.

The contributions to the total refraction by individual layers for zenith distances of 60° and 85° in both the wet and the dry atmospheric model are given in Table 10. For $z = 60^\circ$, the layers are 1 km thick up to 5 km, and 5 km thick from 5 to 30 km. For $z = 85^\circ$, the layers are 1 km thick up to 10 km, and 5 km thick from 10 to 30 km. The contribution by the 5 km layer between 50 and 55 km is also shown for $z = 85^\circ$. The differences between the refractions in the wet and the dry atmospheric models, accumulated down to any level, h, is shown in Figure 6. It is to be noted that from their effective tops (90 km) down to 30 km, both atmospheres give essentially the same amount of accumulated refraction, and the maximum differences at the surface are only $0\overset{''}{.}164$ in the $z = 60^\circ$ case and $1\overset{''}{.}073$ in the $z = 85^\circ$ case. The total refraction computed by numerical integration is given in Table 11 for the wet and the dry cases for zenith distances of 60° and 85°. For the sake of comparison the values of the refraction for these same cases computed from the Pulkova tables and using Willis' method are also presented in this table. In order to

compute the values for the wet atmosphere by these last two methods, the surface water vapor content was taken to be that assumed in the "moist ARDC" case and the surface temperature taken to be that required by the assumption therein of an unchanged pressure profile.

Table 10

REFRACTION IN INDIVIDUAL LAYERS
OF THE ARDC (WET) AND ARDC (DRY) ATMOSPHERIC MODELS

	$z = 60^{\circ}$			$z = 85^{\circ}$	
Layer (km)	Wet	Dry	Layer (km)	Wet	Dry
30 - 25	1".8127	1".8127	55 - 50	0".1432	0".1432
25 - 20	3.8527	3.8526	30 - 25	8.4608	8.4604
20 - 15	8.4748	8.4748	25 - 20	18.8939	18.8933
15 - 10	17.5715	17.5711	20 - 15	43.9055	43.9039
10 - 5	26.0232	26.0161	15 - 10	96.7247	96.7188
5 - 4	6.6989	6.6924	10 - 9	24.6451	24.6432
4 - 3	7.2726	7.2605	9 - 8	27.4151	27.4098
3 - 2	7.8844	7.8623	8 - 7	30.4340	30.4264
2 - 1	8.5347	8.4943	7 - 6	33.7160	33.7018
1 - 0	9.2390	9.1640	6 - 5	37.2736	37.2525
			5 - 4	41.1597	41.1182
			4 - 3	45.3749	45.2972
			3 - 2	49.9676	49.8252
			2 - 1	54.9589	54.6973
			1 - 0	60.4719	59.9802

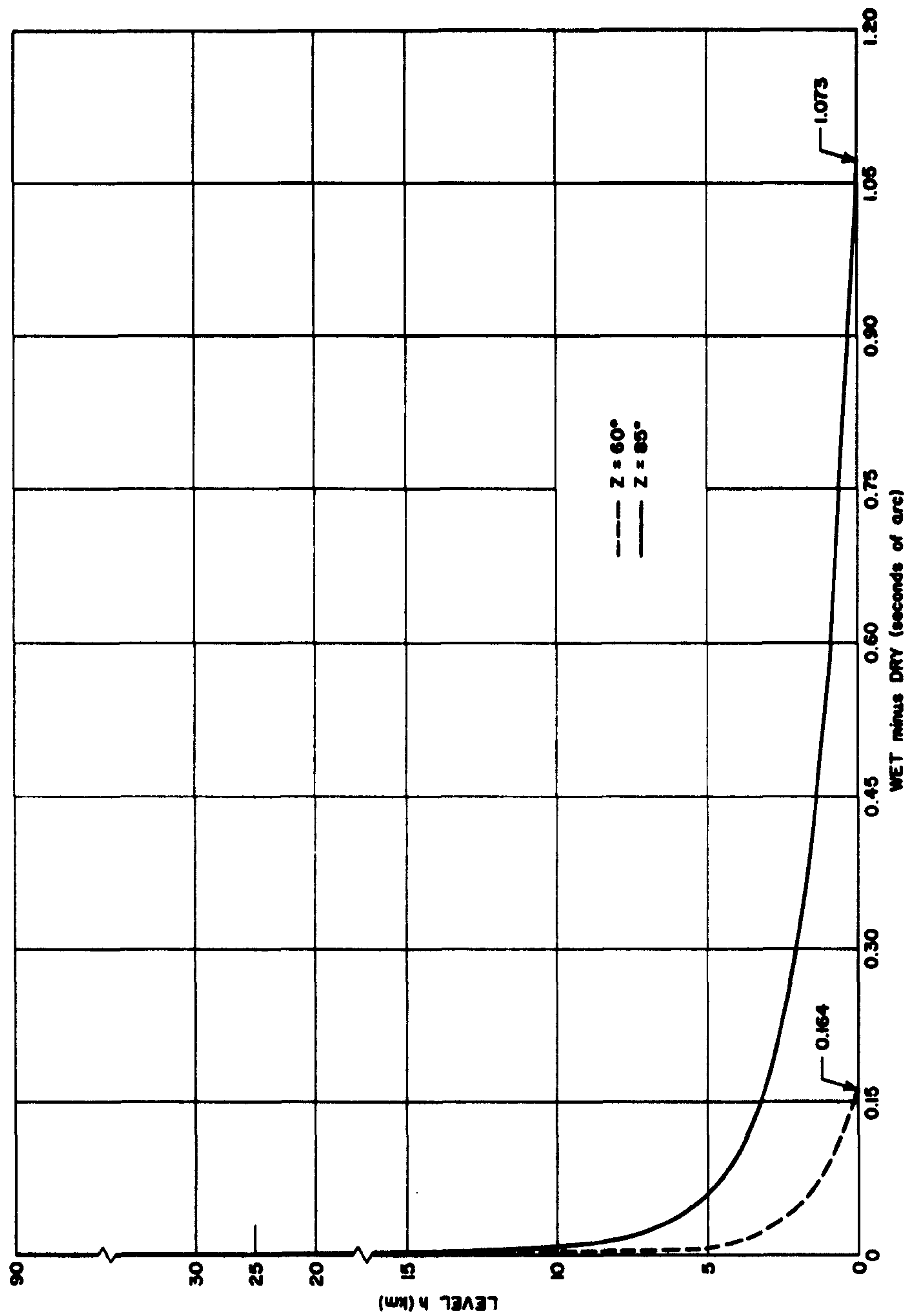

FIG. 6. THE DIFFERENCES BETWEEN THE REFRACTIONS IN THE WET AND THE DRY ARDC ATMOSPHERE MODELS, ACCUMULATED DOWN TO ANY LEVEL h

Table 11

SUMMARY OF REFRACTION CALCULATIONS

Surface Conditions	Wet	Dry	Difference Wet - Dry
t	14.19°C	15°C	
p	760 mm Hg	760 mm Hg	
w	.00464	0	
Method		z = 60°	
ARDC Integration (A)	98".784	98".620	+ 0".164
Willis Tables (W)	98.729	98.563	+ 0.166
Pulkovo Tables (P)	98.675	98.512	+ 0.163
A - W	+ 0".055	+ 0".057	
A - P	+ 0.109	+ 0.108	
W - P	+ 0.054	+ 0.051	
Method		z = 85°	
ARDC Integration (A)	579".887	578".814	+ 1".073
Willis Tables (W)	580.176	579.026	+ 1.150
Pulkovo Tables (P)	579.657	578.512	+ 1.145
A - W	- 0".289	- 0".212	
A - P	+ 0.230	+ 0.302	
W - P	+ 0.519	+ 0.514	

The agreement between the results of the three methods is remarkably good; even in the very demanding case of z = 85° the maximum spread between all three methods is 0".077 in a total refraction of nearly 580". The 85° integration indicates an increase of the wet over the dry refraction of about 0".08 less than the increase indicated by the other two methods, but the actual refraction derived from the integration falls between the Pulkovo and the Willis values in both the wet and dry cases.

It is, of course, important to note that all values calculated

are based upon atmospheric models; one, the ARDC atmosphere being an explicit model, while the other two are implicit in the Pulkovo and Willis tables for computing astronomical refraction. The fact that both the Pulkovo and Willis tables are based upon precise astronomical observations and that, furthermore, these tables can be used in the inverse sense to compute refraction corrections to precise astronomical measurements implies that the implicit atmospheric models associated with these tables approximate exceptionally well the actual atmosphere, at least on the average. The almost negligible divergence of the refraction integration based on the ARDC atmosphere from the Willis or Pulkovo values implies, at the least, that the ARDC atmosphere is itself as good an approximation to the true atmosphere as the other two. This favorable comparison also implies that it may well be possible to obtain even more accurate values of atmospheric refraction by direct calculation, that is, integration, of an atmospheric model representing a suitable modification of the ARDC model, the modification being based upon a limited sounding of the atmosphere at the observing site or (even better) upon an analysis of synoptic weather data for the general locale of the site.

VI. RECOMMENDATIONS

There are now available several possible ways of calculating with considerable precision the angular refraction in a vertical plane caused by the earth's atmosphere. These range from the astronomical-observation-based Pulkova Tables used in one or another of their successively improved forms by positional astronomers for over a century, to integration through the idealized ARDC atmosphere which has been derived from balloon, rocket and satellite soundings. Viewed in this light, the really remarkable agreement exhibited by the selected cases we have treated in this report and our earlier one, is on the one hand assurance that any one of the several diverse approaches provides accuracy sufficient for the vast majority of present needs, and on the other hand a strong indication that we are indeed also in a favorable position to attempt some further gains in accuracy through closer adaptation of the chosen model atmosphere to the actual conditions obtaining along the ray path.

1. We recommend that a careful and extensive intercomparison be made of the refraction values yielded by the most recent ARDC, Tropical, and Arctic Winter atmosphere models and the Pulkova, Willis and Astronomische-Geodätische Jahrbuch (1952) tables. This intercomparison will serve two objectives. First, it will provide a means for assessing the magnitude of the error which can occur in the deduced position of a low satellite as a result of an incompatibility of the measured

surface refractive index with the true total atmospheric refraction at the instant, an error which we have reason to believe is, in practice, considerably less in magnitude than some investigators have suggested. Second, it will provide a basis for the study of refractions in the "hybrid" models described in Recommendation 5.

2. We recommend that the basic Pulkova tables be carefully smoothed to one more decimal place and the auxiliary formulas and corresponding tables be extended by one decimal through utilization of the slightly improved physical constants (e.g. the refractive index of water vapor) which have become available since the most recent edition of the Pulkova Tables. This will reduce the number of "sensitive" cases (for low satellites or very large zenith angles) where round-off error can render the computed refraction slightly inconsistent with the surface refractive index supposed to go with it in the particular tabularly-defined model. A similar smoothing (to remove round-off error but <u>not</u> to affect the known physical discontinuities incorporated in the tabulated models) should be carried out on the ARDC and related models for the same purpose, to the extent deemed necessary to produce, for the sensitive cases, the maximum accuracy of model representation attainable.

3. We recommend that an integrated study be made of the systematic corrections to the standard astronomical refraction tables. Such corrections have already been derived at a number of astronomical observatories. These corrections should be correlated with the results of the intercomparisons

obtained in Recommendation 1. These correlations should then make it possible either to establish the currently used tables (with the small modifications cited) as providing adequate accuracy singly or in judicious combination for all presently envisioned needs, or to derive (using them as a guide) methods for adjusting chosen physical model atmospheres singly or in combination, to local conditions, along the lines suggested by our work on the introduction of water vapor into the ARDC.

4. We recommend that, along with this study involving the systematic "observatory corrections," an investigation be undertaken of selected portions of the vast amount of data on the more or less random variations in refraction available for the group of observatories chosen under Recommendation 3. Such an investigation should result in a meaningful assessment of the extent of the scatter of refraction effects in actual precise observations and the character of the scatter in relation, for example, to the instantaneous surface index of refraction. This in turn should indicate the range of incompatibility of that refractive index and the instantaneous total atmospheric refraction actually existent.

5. We recommend that a thorough investigation be made of the improvement obtainable in "instantaneous" total atmospheric refraction values at a station by appropriately combining segments of the ARDC, Tropical, and Arctic Winter physical models, suitably adjusted for local variation in the governing parameters. These atmospheric segments should

be combined by techniques similar to those described in the body of this report. The basis for the selection of the atmospheric segments would be the air-mass data determined from synoptic weather charts for the surface and upper air relevant to that particular portion of the earth's atmosphere traversed by the ray being studied. We feel that this is the most promising mode of attack for obtaining highly precise, instantaneous values of the refraction which are truly compatible with the surface parameters.

6. We recommend initiation of a study of horizontal refraction in the earth's atmosphere based upon both the observatory data mentioned above and the geometry of the azimuthal discontinuities at air-mass fronts, which violate, spatially and temporally, the assumption of a spherically stratified atmosphere basic to the standard formulations for vertical refraction. Such a study could be pursued most efficiently as an adjunct to the very important one outlined in Recommendation 5, since it would involve much of the same information and some of the same lines of thought.

7. We recommend that the effect of the variation of refractive index along the ray path, in terms of both geometrical path-length and transit time, for the case of trilateration on a satellite, be carefully studied by integration methods essentially paralleling those we have used. The three atmospheres (ARDC, Tropical, Arctic Winter) should be taken as the basic models. Two problems should be given special attention: (1) adaptation of the atmospheric model to take account of the air masses

present and (2) the modification due to introduction of water vapor. The latter problem is particularly important at radio frequencies. To the first order, at least, such a study is step-by-step an analog of the angular deviation case; the integration would presumably have to be extended to greater heights because of the cumulative effect on the transit time, of the extensive regions where the index is not quite unity but so nearly constant that no measurable angular effect arises.

8. We recommend that careful consideration be given to the question of exactly how the index of refraction of humid air is related to the wave length of the radiation traversing it. In the visible and near infra-red the index decreases slowly with increasing water vapor content, but at radio wave lengths the index increases rapidly with increasing water vapor. It is therefore possible that at one (or more) intermediate wave length the effect is zero, that is, that the refractive index is independent of the amount of water vapor present. If this should be the case, then a laser (or a microwave maser) operating at that wave length would be of enormous potential value.

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REFERENCES

1. The Franklin Institute Laboratories, "Research in Surveying, Mapping and Geodesy," Q-A2217-6, September 4, 1960 to March 3, 1961.

2. Minzner, R. A., Champion, K. S. W., and Pond, H., "The ARDC Model Atmosphere, 1959," Air Force Surveys in Geophysics, No. 115, AFCRC, 1959.

3. Willis, J. E., "A Determination of Astronomical Refraction from Physical Data," Transactions, American Geophysical Union, 22, 324, 1941.

4. Refraction Tables of the Pulkovo Observatory, Fourth Edition, Academy of Sciences Press, Moscow, 1956.

5. Barrell, H., and Sears, J. E. "Refraction and Dispersion of Air in the Visible Spectrum; Transactions, Royal Society of London, A238, 1, 1939.

6. Allen, C. W., Astrophysical Quantities, University of London, The Athlone Press, 1955.

7. Murcray, D. G., Murcray, F. H., and Williams, W. J., "Distribution of Water Vapor in the Stratosphere as Determined from Infrared Absorption Measurements," University of Denver, January, 1961.

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